APPLICATIONS INVOLVING TRIGONOMETRIC FUNCTIONS AND GRAPHS

1 A Ferris wheel with a radius of 35 m rotates once every 90 seconds. Passengers enter a cabin at the loading point, which is 2 m above level ground. The height, $h(t)$ metres, above the ground is given by

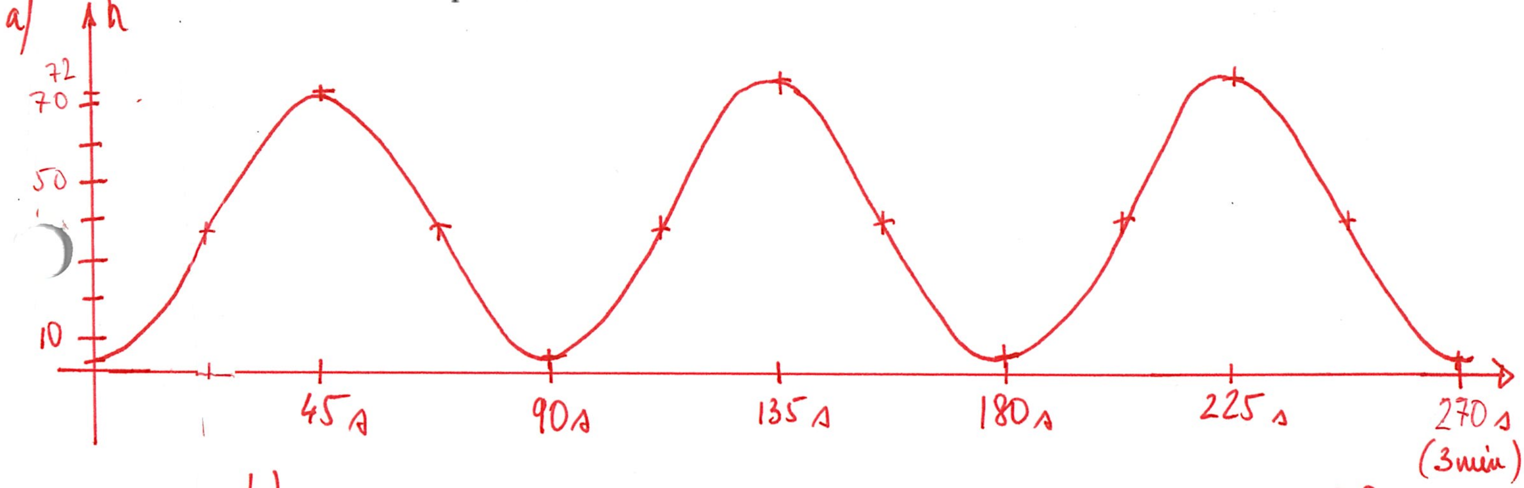
$h(t) = -35\cos\frac{\pi t}{45} + 37$, where t seconds is the time after leaving the lowest level.

(a) Sketch the graph of $h(t)$ for a 3-minute period after a cabin passes the loading point and state the coordinates of the h intercept.

(b) What is a cabin's height above the ground in metres after 30 seconds? Give your answer correct to 1 decimal place.

(c) After how many seconds will the cabin will be 40 m above the ground for the first time? Give your answer correct to two decimal places.

a)



b) at $t = 30$ $\quad h(30) = -35\cos\frac{\pi \times 30}{45} + 37$

$h(30) = -35\cos\frac{2\pi}{3} + 37$

$h(30) = -35 \times \left(-\frac{1}{2}\right) + 37 = 54.5\text{m}$

c) We look for t so that $h(t) = 40$

$-35\cos\left(\frac{\pi t}{45}\right) + 37 = 40$

$-35\cos\left(\frac{\pi t}{45}\right) = 3$

$+\cos\left(\frac{\pi t}{45}\right) = \frac{-3}{35}$

So $\frac{\pi t}{45} = \cos^{-1}\left(\frac{-3}{35}\right)$ or $t = \frac{45}{\pi}\cos^{-1}\left(\frac{-3}{35}\right) \approx 23.73\text{s}$

(Remember we use radians).

4 The voltage, V volts, supplied by an electrical outlet is described by the sine function $V = k\sin at + c$, where k, a and c are constants, t is measured in seconds and $k > 0$.

(a) If the voltage, V, oscillates between −240 volts and 240 volts, find the value of k and c.
(b) If it has a frequency of 50 cycles per second, find the value of a.
(c) Determine the voltage (V) involving a transformation of the sine function.

a) $c=0$ $V = k \sin at$ k must be $240\,V$ $V = 240 \sin at$

b) $\sin(at)$ varies between -1 and 1 every $\frac{1}{50}$ s.

so $a \times \frac{1}{50} = 2\pi$ so $a = 100\pi$

c) $V = 240 \sin(100\pi t)$

5 The average daily maximum and minimum temperatures (in °C) for Melbourne are given in the table. Time, t, is measured in months, with $t = 0$ representing 1 January.
The average temperatures for Melbourne are as follows:

Temperature	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Maximum Temperature	26°	26°	24°	20°	17°	14°	13°	15°	17°	20°	22°	24°
Minimum Temperature	14°	14°	13°	11°	8°	7°	6°	7°	8°	9°	11°	13°

Write trigonometric models, involving transformation of the sine function, that give the average daily maximum and minimum temperatures for Melbourne as functions of time, t.

(a) Average daily maximum temperature. **(b)** Average daily minimum temperature.

Max = 26
Min = 13 so amplitude is $\frac{13}{2} = 6.5$
So it'll look like
$T = 6.5 \cos[a(t+\alpha)] + 19.5$ actually the max is at $t = 1$
so $T = 6.5 \cos\left[\frac{1}{2}(t+2)\right] + 19.5$

$T = 4 \sin\left[\frac{1}{2}(t+2)\right] + 10$

6 Scientists monitoring a population of penguins on Heard Island found that the number of penguins varies between a high of 900 at the start of the year and a low of 700 on 30 June and is approximated by the cosine function $P(t) = k\cos at + c$, where t is the number of months since the start of the year, and k, a and c are constants.

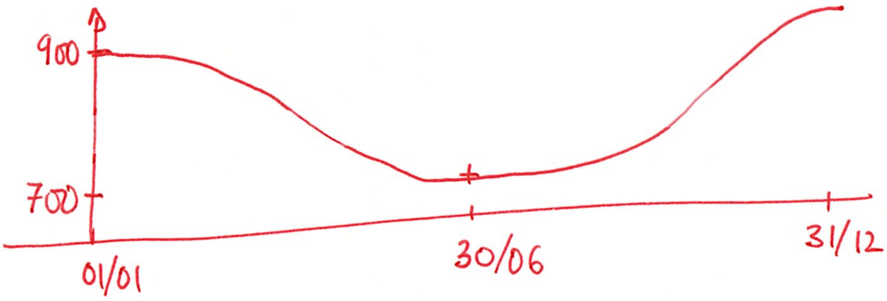
(a) Sketch a graph of the penguin population over a whole year.

(b) Find the values of k, a and c for the function.

(c) According to the population function, how many penguins will there be on Penguin Island on 31 March? 20 June? 23 October?

a) 1^{st} Jan : 900 penguins   30 June = 700 penguins

$P(t) = k\cos(at) + c$



b) $k = 100$ $c = 800$

$P(t) = 100\cos at + 800$ $a = \frac{\pi}{6}$

$P(t) = 100\cos\left(\frac{\pi t}{6}\right) + 800$

c) $P(3) = 100\cos\left(\frac{\pi}{6} \times 3\right) + 800 = 800$

$P\left(5\frac{2}{3}\right) = 100\cos\left[\frac{\pi}{6} \times \left(5\frac{2}{3}\right)\right] + 800 = 702$

23 october is $9 + \frac{23}{30} = 9\frac{23}{30}$

$P(23\text{ oct}) = 100\cos\left(\frac{\pi}{6} \times 9\frac{23}{30}\right) + 800 = 839$

7 The tide at a point on the WA coast can be modelled using the equation $y = k\cos nt$. At Cable Beach in WA, over two consecutive days, the average difference between high and low tides is 9.0 metres and the average time between high tide and low tide is 6.1 hours.

(a) What is the amplitude of the tide function at Cable Beach?
(b) How much time passes between successive high tides (i.e. the period) and what is the value of n?
(c) Use this information to obtain the tide function and draw its graph.
(d) If the depth of water at low tide is 0.5 metres, what is the depth of the water 1 hour after low tide?

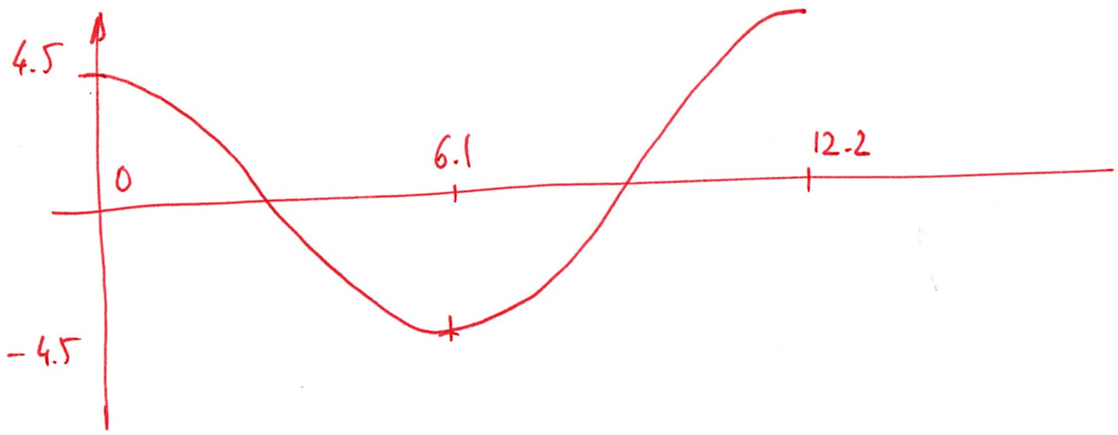
a) Amplitude is $9/2 = 4.5$ m

b) Period is $6.1 \times 2 = 12.2$ hours.

so $n \times 12.2 = 2\pi$

so $n = \frac{2\pi}{12.2} = \frac{\pi}{6.1} = \frac{10\pi}{61}$

c) $y = 4.5\cos\left(\frac{10\pi}{61}t\right)$



d) At low tide, $t = 6.1$.

For $t = 6.1$, $y = 4.5\cos\left(\frac{10\pi}{61} \times 6.1\right) = 4.5\cos\pi = -4.5$

But in fact, the depth of water at that time is 0.5 m,

So there's a difference of 5 m between the curve and the depth of water (i.e. we need to increase the value by 5 m)

One month after low tide, $t = 1 + 6.1 = 7.1$

$y = 4.5\cos\left(\frac{10\pi}{61} \times 7.1\right) = -3.92$ so, with 5 m added, it becomes $5 + (-3.92) = 1.08$ m