

PROBABILITY

Probability is the area of mathematics concerned with the likelihood of particular random events.

examples:

- when rolling a dice, the likelihood of getting a 2
- when throwing a coin, the likelihood of a tail
- the likelihood that a team wins the next game if they have won all previous games
- the likelihood of winning at lotto.

PROBABILITY - DEFINITIONS

A **trial** is a single occurrence of a chance experiment

example: a single roll of a dice

An **outcome** is the result of an experiment or game. example: when two coins are tossed, one possible outcome is head-tail

The **sample space** is the set of all outcomes.

example: when two coins are tossed, the sample space has 4 outcomes (HH, HT, TH, TT)

PROBABILITY - DEFINITIONS

Equally likely outcomes are outcomes that have the same chance of occurring

example: when two coins are tossed, one possible outcome is head-tail

An **event** is a group of one or more outcomes.

example: when two coins are tossed, the event of both coins being the same consists of 2 outcomes (HH, TT)

THEORETICAL PROBABILITY

If all outcomes in a sample space are equally likely, the **theoretical probability** of an event occurring is:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

A probability can be expressed as a fraction or decimal (ranging from 0 to 1) or a percentage (from 0% to 100%).

A event certain to occur has a probability of 1 (0 for an impossible event).

EXAMPLE OF CALCULATION - PROBABILITY

A letter is chosen from the word TELEVISION. Find the probability that the letter is:

a a V

b an E

c not an E

d either an E or a V

a $P(V) = \frac{1}{10} (= 0.1)$

$$P(V) = \frac{\text{number of Vs}}{\text{total number of letters}}$$

b $P(E) = \frac{2}{10}$
 $= \frac{1}{5} (= 0.2)$

There are 2 Es in the word TELEVISION.

Simplify the fraction.

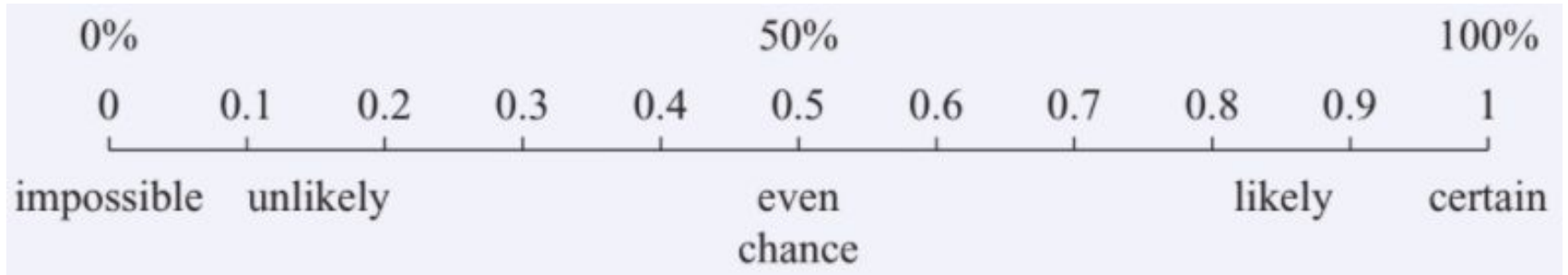
c $P(\text{not an E}) = \frac{8}{10}$
 $= \frac{4}{5} (= 0.8)$

If there are 2 Es in the word TELEVISION with 10 letters, then there must be 8 letters that are not E.

d $P(\text{an E or a V}) = \frac{3}{10} (= 0.3)$

The number of letters that are either E or V is 3.

WORDS THAT DESCRIBE PROBABILITY



EXPERIMENTAL PROBABILITY

Experimental **probability** is calculated in the same way as theoretical probability, but uses the results of an experiment.

$$P(\text{event}) = \frac{\text{number of favourable results}}{\text{total number of trials}}$$

Example: throwing a coin 20 times, and noting how many tails you got.

An experiment involves tossing three coins and counting the number of heads. Here are the results after running the experiment 100 times.

Number of heads	0	1	2	3
Frequency	11	40	36	13

Find the experimental probability of obtaining:

- a** zero heads
- b** two heads
- c** fewer than two heads
- d** at least one head

SOLUTION

a $P(0 \text{ heads}) = \frac{11}{100}$
 $= 0.11$

b $P(2 \text{ heads}) = \frac{36}{100}$
 $= 0.36$

c $P(\text{fewer than 2 heads}) = \frac{11 + 40}{100}$
 $= \frac{51}{100}$
 $= 0.51$

d $P(\text{at least 1 head}) = \frac{40 + 36 + 13}{100}$

EXPLANATION

$P(0 \text{ heads}) = \frac{\text{number of times 0 heads are observed}}{\text{total number of trials}}$

$P(2 \text{ heads}) = \frac{\text{number of times 2 heads are observed}}{\text{total number of trials}}$

Fewer than 2 heads means to observe 0 or 1 head.

At least 1 head means that 1, 2 or 3 heads can be observed. This is the same as $1 - P(0 \text{ heads})$.

BASIC PROBABILITY

Let E be an event. The **complementary event** is noted $\overline{E}$

If $P(E)$ is the probability of the event E occurring, then $P(\overline{E})$ is the probability of E NOT occurring.

$$P(E) + P(\overline{E}) = 1$$

$$\text{or } P(\overline{E}) = 1 - P(E)$$

BASIC PROBABILITY

Example: when throwing 3 dices, probability of getting at least one 6.

$$P(\overline{E}) = 1 - P(E)$$

$$P(\text{getting at least one 6}) = 1 - P(\text{not getting one 6})$$

$$P(\text{getting at least one 6}) = 1 - \left(\frac{5}{6}\right)^3$$

$$P(\text{getting at least one 6}) = 91/216$$