COMPLETING THE SQUARE

In the expansion $(x+6)^2 = x^2 + 12x + 36$, the constant term 36 is half the coefficient of x squared: $\left(\frac{12}{2}\right)^2 = 36$.

In the expansion $(x-5)^2 = x^2 - 10x + 25$, the constant term 25 is half the coefficient of x squared: $\left(\frac{-10}{2}\right)^2 = 25$.

This can be shown for the general expansion $(x-a)^2 = x^2 - 2ax + a^2$ as $\left(\frac{-2a}{2}\right)^2 = a^2$.

Thus, an expression like $x^2 + 6x$ can be made into a perfect square by adding $\left(\frac{6}{2}\right)^2 = 9$ to obtain $x^2 + 6x + 9 = (x+3)^2$.

Example 12

What must be added to each expression to complete the square?

(a) $x^2 + 8x$

(b) $x^2 - 3x$

Solution

(a) $x^2 + 8x$

- Half of 8 is 4
  (This is the value that will go in the brackets.)
- The square of 4 is 16
- Hence 16 must be added

Check: $x^2 + 8x + 16 = (x+4)^2$

(b) $x^2 - 3x$

- Half of −3 is $-\frac{3}{2}$
- The square of $-\frac{3}{2}$ is $\frac{9}{4}$
- Hence $\frac{9}{4}$ must be added

Check: $x^2 - 3x + \frac{9}{4} = \left(x - \frac{3}{2}\right)^2$