PROBLEMS INVOLVING DISPLACEMENT AND VELOCITY

Displacement

When considering the motion of an object, you need to consider the position of the object, how fast it is travelling and the cause of the motion.

The position of an object must be defined relative to a reference point, which is usually its starting point. The position can be described by both its distance and its direction from the starting point. This is a vector quantity, called **displacement**. On the other hand, if you are only concerned with how far the object has travelled then the direction can be ignored, so you can consider the **distance** travelled by the object. The international system of units (SI) uses the metre (m) as the standard unit for displacement and for distance. Other common SI units include the centimetre (cm), millimetre (mm) and kilometre (km).

For example, if you walk around a square park of side length 500 m, the distance you travel is 2000 m. However, as you arrive back at your starting point, your displacement is zero.

Velocity

The rate at which the displacement of an object changes with respect to time can be described by the vector quantity **velocity**. As a vector quantity, velocity is defined by its magnitude and direction. The magnitude of velocity is the scalar quantity **speed**. The SI standard unit for velocity and speed is metres per second (m/s or $\mathrm{m\,s^{-1}}$). Another common unit is kilometres per hour (km/h or $\mathrm{km\,h^{-1}}$).

The average velocity of an object between two positions is defined as:

$$\text{average velocity} = \frac{\text{change in position}}{\text{time taken}} = \frac{\text{displacement}}{\text{time}}$$

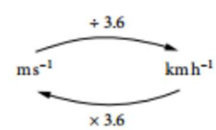
On the other hand, the average speed between two points is defined as:

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

Note: Speed may need to be converted from $\mathrm{m\,s^{-1}}$ to $\mathrm{km\,h^{-1}}$ or from $\mathrm{km\,h^{-1}}$ to $\mathrm{m\,s^{-1}}$:

For example: $90\,\mathrm{km\,h^{-1}} = 90 \times 1000 \div (60 \times 60) = 90 \div 3.6 = 25\,\mathrm{m\,s^{-1}}$

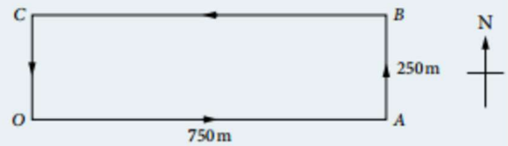
and $15\,\mathrm{m\,s^{-1}} = 15 \times 3.6 = 54\,\mathrm{km\,h^{-1}}$.



- *Displacement* is the change in position of an object relative to its starting point. It is a vector quantity with both magnitude and direction.
- *Distance* is how far an object has travelled. It is a scalar quantity with magnitude only.
- *Velocity* is the rate at which the displacement of an object is changing with respect to time. It is a vector quantity with both magnitude and direction.
- *Speed* is the magnitude of the object's velocity. It is a scalar quantity with magnitude only.

Example 1

Marli is going for a walk around the block. She starts at O and moves to point A and then to point B, where she stops to talk to a friend.



Determine:

(a) the distance Marli travels from O to B

(b) Marli's displacement from O to B, correct to one decimal place.

If Marli takes 10 minutes to reach point B, determine:

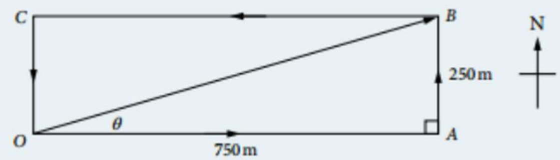
(c) her average speed in metres per second, correct to one decimal place

(d) her average velocity in metres per second, correct to one decimal place.

Solution

(a) Distance travelled from O to point A to point $B = 750 + 250 = 1000\,\text{m}$

(b) Using Pythagoras' theorem to find the distance between the starting and finishing points:



$$|\overline{OB}| = \sqrt{750^2 + 250^2}$$
$$= 790.6\,\text{m}$$

Direction of motion: $\theta = \tan^{-1}\left(\frac{250}{750}\right)$
$$= 18.4°$$

Bearing is N(90 – 18.4)°E = N71.6°E

Marli's displacement is 790.6 m from O in a direction of N71.6°E.

(c) $10\,\text{min} = 600\,\text{s}$

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{1000}{600} = 1.7\,\text{m}\,\text{s}^{-1}$$

(d)
$$\text{average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{790.6}{600}$$
$= 1.3\,\text{m}\,\text{s}^{-1}$ in the direction N71.6°E.

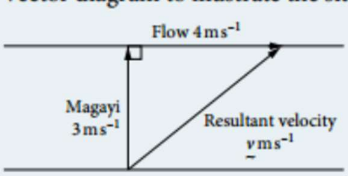
Finding resultant velocity

Example 2

Magayi can swim in still water at a rate of $3.0\,\text{m}\,\text{s}^{-1}$. If she swims in a river that is flowing at $4.0\,\text{m}\,\text{s}^{-1}$ and keeps her direction (with respect to the water) perpendicular to the flow, find the magnitude of her velocity with respect to the riverbank.

Solution

Vector diagram to illustrate the situation:



Using Pythagoras' theorem to find the magnitude of the resultant velocity $\underset{\sim}{v}$:

$$|\underset{\sim}{v}| = \sqrt{3^2 + 4^2}$$
$$= 5\,\text{m}\,\text{s}^{-1}$$

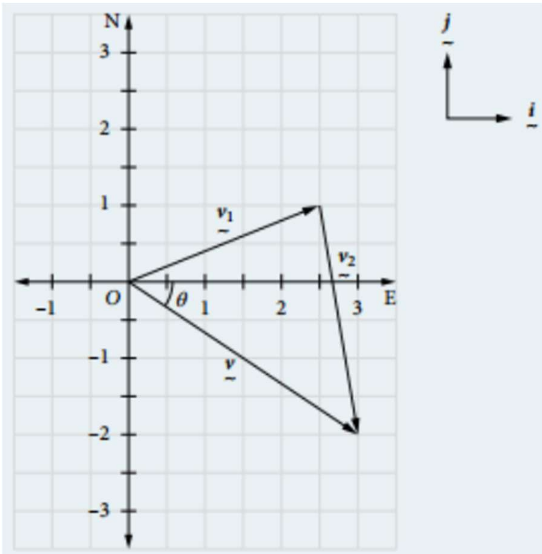
Finding resultant velocity using component form

Example 3

Let $\underset{\sim}{i}$ and $\underset{\sim}{j}$ be unit vectors in the directions of east and north, respectively. Pravat is swimming in the ocean and his velocity relative to the water $\underset{\sim}{v}_1$ m s^{-1} is given by the vector $\underset{\sim}{v}_1 = 2.5\underset{\sim}{i} + 1.0\underset{\sim}{j}$. The ocean's current has a velocity $\underset{\sim}{v}_2$ m s^{-1} where $\underset{\sim}{v}_2 = 0.5\underset{\sim}{i} - 3.0\underset{\sim}{j}$. Find the magnitude and direction of Pravat's resultant velocity $\underset{\sim}{v}$ m s^{-1}, correct to one decimal place.

Solution

Vector diagram to illustrate the situation:



Adding vectors to find the resultant velocity:

Adding vectors to find the resultant velocity:

$$\underset{\sim}{v} = \underset{\sim}{v}_1 + \underset{\sim}{v}_2$$
$$= 2.5\underset{\sim}{i} + 1.0\underset{\sim}{j} + 0.5\underset{\sim}{i} - 3.0\underset{\sim}{j}$$
$$= 3.0\underset{\sim}{i} - 2.0\underset{\sim}{j}$$

$$|\underset{\sim}{v}| = \sqrt{(3.0)^2 + (-2.0)^2}$$
$$= \sqrt{13.0}$$
$$= 3.6 \text{ m s}^{-1}$$

θ is an acute angle in the diagram, so use positive values for side lengths.

$$\theta = \tan^{-1}\left(\frac{2.0}{3.0}\right)$$
$$= 33.7°$$

The direction bearing: $90 + 33.7 = 123.7°$T.

Pravat is swimming at 3.6 m s^{-1} in a direction of 123.7°T.