INTEGRALS RESULTING IN LOGARITHMIC FUNCTIONS

1 Find the primitive of the following:

(a) $\frac{2}{x}$ (b) $\frac{1}{x+1}$ (c) $\frac{2}{2x+1}$ (d) $\frac{x}{x^2-4}$

a) $\int \frac{2}{x}\,dx = 2\int \frac{1}{x}\,dx = 2\ln|x| + C$

b) $\int \frac{1}{x+1}\,dx = \ln|x+1| + C$

c) $\int \frac{2}{2x+1}\,dx = \ln|2x+1| + C$

d) $\int \frac{x}{x^2-4}\,dx = \frac{1}{2}\int \frac{2x}{x^2-4}\,dx = \frac{1}{2}\ln|x^2-4| + C$

(e) $\frac{1}{2x-1}$ (f) $\frac{e^x}{4+e^x}$ (g) $\frac{x^3}{x^4+1}$ (h) $\frac{e^{2x}}{4-e^{2x}}$

e) $\int \frac{1}{2x-1}\,dx = \frac{1}{2}\int \frac{2}{2x-1}\,dx = \frac{1}{2}\ln|2x-1| + C$

f) $\int \frac{e^x}{4+e^x}\,dx = \ln|4+e^x| + C$

g) $\int \frac{x^3}{x^4+1}\,dx = \frac{1}{4}\int \frac{4x^3}{x^4+1}\,dx = \frac{1}{4}\ln|x^4+1| + C$

h) $\int \frac{e^{2x}}{4-e^{2x}}\,dx = \int \frac{-e^{2x}}{e^{2x}-4}\,dx = -\int \frac{e^{2x}}{e^{2x}-4}\,dx$

$= -\frac{1}{2}\int \frac{2e^{2x}}{e^{2x}-4}\,dx = -\frac{1}{2}\ln|e^{2x}-4| + C$

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2 If $\frac{dy}{dx} = \frac{1}{x}$ and $y = 0$ where $x = 2$, then the correct expression for y in terms of x is:

A $y = \log_e x - 2$ **B** $y = \frac{1}{2}\log_e x$ **C** $y = \log_e\left(\frac{x}{2}\right)$ **D** $y = 2\log_e x$

$y = \ln|x| + C$. But $y = 0$ when $x = 2$ so $0 = \ln|2| + C$

So $C = -\ln|2|$ $y = \ln|x| - \ln 2 = \ln\left(\frac{x}{2}\right)$ $\boxed{C}$

3 The gradient of a curve at any point is $\frac{2}{2x+1}$ and the curve passes through the point $(1, \log_e 3)$. Find the equation of the curve.

$\int \frac{2}{2x+1}\,dx = \ln|2x+1| + C$ so $y = \ln|2x+1| + C$

when $x = 1$ $y = \ln 3$ so $\ln 3 = \underbrace{\ln|2\times1+1|}_{=\ln 3} + C$

So $C = 0$

$y = \ln|2x+1|$

4 Find the rule that defines $f(x)$ given that $f'(x) = \frac{x}{x^2+9}$ and $f(0) = \log_e 3$. Indicate whether each statement below is a correct or incorrect step in the solution of this problem.

(a) $f(x) = \int \frac{x}{x^2+9}dx$ correct **(b)** $f(x) = \frac{1}{2}\ln(x^2+9) + C$ correct **(c)** $C = 2\ln 3$ incorrect **(d)** $f(x) = \frac{1}{2}\ln(x^2+9)$ correct

$f(x) = \int \frac{x}{x^2+9}\,dx = \frac{1}{2}\int \frac{2x}{x^2+9}\,dx = \frac{1}{2}\ln|x^2+9| + C = \frac{1}{2}\ln(x^2+9) + C$

$f(0) = \ln 3$ so $\ln 3 = \frac{1}{2}\ln(0^2+9) + C = \frac{1}{2}\ln 9 + C = \frac{1}{2}\ln 3^2 + C = \ln 3 + C$

So $C = 0$

$\boxed{C}$ incorrect

$\boxed{D}$ correct

5 Write the derivative of $\log_e(\cos x)$ and hence find the primitive function of $\tan x$.

$\frac{d}{dx}\left(\ln(\cos x)\right) = \frac{1}{\cos x} \times (-\sin x) = -\tan x$

So $\int \tan x\,dx = \ln|\cos x| + C$

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6 Differentiate: **(a)** 2^x **(b)** $x+10^x$ **(c)** e^x+5^x **(d)** 5^{x^2} **(e)** $a^{\sqrt{x}}$

a) $\frac{d}{dx}(2^x) = \frac{d}{dx}\left(e^{x\ln 2}\right) = e^{x\ln 2} \times \ln 2 = 2^x \times \ln 2$

b) $\frac{d}{dx}\left(x+10^x\right) = 1 + \frac{d}{dx}\left(10^x\right) = 1 + \ln 10 \times 10^x$

c) $\frac{d}{dx}\left(e^x+5^x\right) = e^x + \ln 5 \times 5^x$

d) $\frac{d}{dx}\left(5^{x^2}\right) = \ln 5 \times 5^{x^2} \times 2x = 2\ln 5\, x \times 5^{x^2}$

e) $\frac{d}{dx}\left(a^{\sqrt{x}}\right) = \ln a \times a^{\sqrt{x}} \times \left(\sqrt{x}\right)'$

$\qquad = \ln a \times a^{\sqrt{x}} \times \left(x^{1/2}\right)'$

$\qquad = \ln a \times a^{\sqrt{x}} \times \frac{1}{2} x^{-1/2}$

$\qquad = \frac{\ln a \times a^{\sqrt{x}}}{2\sqrt{x}}$

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7 Find: (a) $\int 3^x\,dx$ (b) $\int\left(x+10^x\right)dx$ (c) $\int\left(\frac{1}{x}+1+e^x+a^x\right)dx$

a) $\int 3^x\,dx = \dfrac{3^x}{\ln 3} + C$

b) $\int (x+10^x)\,dx = \dfrac{x^2}{2} + \dfrac{10^x}{\ln 10} + C$

c) $\int\left(\frac{1}{x} + 1 + e^x + a^x\right)dx = \ln|x| + x + e^x + \dfrac{a^x}{\ln a} + C$