1 Solve for $0 \le x \le 2\pi$:

(a) $\sin(2x) = 0.5 = \sin\frac{\pi}{6}$

$2x = (-1)^n \times \frac{\pi}{6} + n\pi$

$x = (-1)^n \frac{\pi}{12} + n\frac{\pi}{2}$

* for $n=0$ $\boxed{x = \frac{\pi}{12}}$

* for $n=1$ $\boxed{x = \frac{5\pi}{12}}$

* for $n=2$ $\boxed{x = \frac{13\pi}{12}}$

* for $n=3$ $\boxed{x = \frac{17\pi}{12}}$

$n=4$ is outside $[0, 2\pi]$

(b) $\cos(3x) = 1 = \cos 0$

$3x = \pm 0 + 2n\pi$

$x = \frac{2n\pi}{3}$

* for $n=0$ $\boxed{x=0}$

* for $n=1$ $\boxed{x = \frac{2\pi}{3}}$

* for $n=2$ $\boxed{x = \frac{4\pi}{3}}$

* for $n=3$ $\boxed{x = 2\pi}$

$n=4$ is outside $[0, 2\pi]$

(c) $\tan(2x) = 1 = \tan\frac{\pi}{4}$

$2x = \frac{\pi}{4} + n\pi$

$x = \frac{\pi}{8} + n\frac{\pi}{2}$

* $n=0$ $\boxed{x = \pi/8}$

* $n=1$ $\boxed{x = \frac{5\pi}{8}}$

* $n=2$ $\boxed{x = \frac{9\pi}{8}}$

* $n=3$ $\boxed{x = \frac{13\pi}{8}}$

$n=4$ gives results outside of $[0, 2\pi]$

2 Solve for $-\pi \le x \le \pi$:

(a) $\tan\left(\frac{x}{2}\right) = \frac{1}{\sqrt{3}} = \frac{1/2}{\sqrt{3}/2} = \tan\frac{\pi}{6}$

$\frac{x}{2} = \frac{\pi}{6} + n\pi$

$x = \frac{\pi}{3} + 2n\pi$

For $n=0$ $\boxed{x = \frac{\pi}{3}}$

all other solutions are outside $[-\pi, \pi]$

(b) $\sin\left(\frac{x}{3}\right) = -\frac{\sqrt{3}}{2} = \sin\frac{4\pi}{3}$

$\frac{x}{3} = \frac{4\pi}{3}(-1)^n + n\pi$

$x = (-1)^n \times 4\pi + n3\pi$

for $n=1$

$\boxed{x = -\pi}$

all other solutions are outside $[-\pi, \pi]$

(c) $\cos\left(\frac{x}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \cos\frac{\pi}{4}$

$\frac{x}{4} = \pm\frac{\pi}{4} + 2n\pi$

$x = \pm\pi + 8n\pi$

For $n=0$

$\boxed{x = \pm\pi}$

No other solutions.

3 Solve, for $0 \le x \le 2\pi$:

(a) $2\cos\left(x-\frac{\pi}{6}\right)=1$

$$\cos\left(x-\frac{\pi}{6}\right)=\frac{1}{2}=\cos\frac{\pi}{3}$$

$$x-\frac{\pi}{6}=\pm\frac{\pi}{3}+2n\pi$$

$$x=\frac{\pi}{6}\pm\frac{\pi}{3}+2n\pi$$

* $n=0$ $\boxed{x=\frac{\pi}{2}}$

* $n=1$ $\boxed{x=\frac{11\pi}{6}}$

and that's it. (other ~~solutions~~ solutions are outside of the interval $[0,2\pi]$

(b) $\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=1$

$$\sin\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}=\sin\left(\frac{\pi}{4}\right)$$

$$x+\frac{\pi}{4}=(-1)^n\frac{\pi}{4}+n\pi$$

$$x=(-1)^n\frac{\pi}{4}-\frac{\pi}{4}+n\pi$$

* for $n=0$ $\boxed{x=0}$

* for $n=1$ $\boxed{x=\frac{\pi}{2}}$

* for $n=2$ $\boxed{x=2\pi}$

Other solutions are outside of the interval $[0,2\pi]$

(c) $2\cos\left(x-\frac{\pi}{3}\right)=\sqrt{3}$

$$\cos\left(x-\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}=\cos\pi/6$$

$$x-\frac{\pi}{3}=\pm\frac{\pi}{6}+2n\pi$$

$$x=\pm\frac{\pi}{6}+\frac{\pi}{3}+2n\pi$$

* for $n=0$

$\boxed{x=\pi/6}$ and $\boxed{x=\frac{\pi}{2}}$

* for $n=1$ Solutions are outside of the interval $[0,2\pi]$ and other solutions are outside of the interval $[0,2\pi]$

4 Solve, for $-\pi \leq x \leq \pi$:

(a) $\sqrt{2}\cos 2x = 1$

$\cos 2x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \cos\frac{\pi}{4}$

$2x = \pm\frac{\pi}{4} + 2n\pi$

$x = \pm\frac{\pi}{8} + n\pi$

* for $n=0$

$\boxed{x = -\frac{\pi}{8} \text{ and } x = \frac{\pi}{8}}$

* for $n=1$

$\boxed{x = \frac{7\pi}{8}}$

* for $n=-1$ $\boxed{x = -\frac{7\pi}{8}}$

other solutions are outside of $[-\pi, \pi]$

(b) $\cos\left(2x - \frac{\pi}{2}\right) = 1 = \cos 0$

$2x - \frac{\pi}{2} = \pm 0 + 2n\pi$

$2x = \frac{\pi}{2} + 2n\pi$

$x = \frac{\pi}{4} + n\pi$

* $n=0$ $\boxed{x = \frac{\pi}{4}}$

* $n=1$ solutions are outside of $[-\pi, \pi]$

* $n=-1$ $\boxed{x = -\frac{3\pi}{4}}$

and that's it.

(c) $\sin\left(2x + \frac{\pi}{6}\right) = -1$

$= \sin\left(\frac{3\pi}{2}\right)$

$2x + \frac{\pi}{6} = (-1)^n\frac{3\pi}{2} + n\pi$

$2x = (-1)^n\frac{3\pi}{2} - \frac{\pi}{6} + n\pi$

$x = (-1)^n\frac{3\pi}{4} - \frac{\pi}{12} + n\frac{\pi}{2}$

* $n=0$ $\boxed{x = \frac{2\pi}{3}}$

* $n=1$ $\boxed{x = -\frac{\pi}{3}}$

other solutions are outside of $[-\pi, \pi]$

5 (a) Solve, for $0 \le x \le 2\pi$, $\sqrt{2}\cos\left(\frac{x}{2}+\frac{\pi}{6}\right)=1.$

$$\cos\left(\frac{x}{2}+\frac{\pi}{6}\right)=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}=\cos\left(\frac{\pi}{4}\right)$$

$$\frac{x}{2}+\frac{\pi}{6}=\pm\frac{\pi}{4}+2n\pi$$

$$\frac{x}{2}=-\frac{\pi}{6}\pm\frac{\pi}{4}+2n\pi$$

$$x=-\frac{\pi}{3}\pm\frac{\pi}{2}+4n\pi$$

* $n=0$ $\boxed{x=\frac{\pi}{6}}$

other solutions are outside of $[0,2\pi]$

(b) Solve, for $-3\pi \le x \le 3\pi$, $\tan\left(\frac{x}{3}-\frac{\pi}{6}\right)=1.$

$$\tan\left(\frac{x}{3}-\frac{\pi}{6}\right)=\tan\left(\frac{\pi}{4}\right)$$

$$\frac{x}{3}-\frac{\pi}{6}=\frac{\pi}{4}+n\pi$$

$$\frac{x}{3}=\frac{\pi}{4}+\frac{\pi}{6}+n\pi$$

$$\frac{x}{3}=\frac{5\pi}{12}+n\pi$$

$$x=\frac{5\pi}{4}+3n\pi$$

* $n=0$ $\boxed{x=\frac{5\pi}{4}}$

* $n=1$ outside the ~~range~~ interval $[-3\pi,+3\pi]$

* $n=-1$ $\boxed{x=-\frac{7\pi}{4}}$

7 Solve over the given domain

(a) $\cos x + 3\sin x = 1$ *for* $0 \le x \le 2\pi$

$\Leftrightarrow \cos x = 1 - 3\sin x$

we square both sides.

So $\cos^2 x = (1 - 3\sin x)^2$ ①

but $\cos^2 x + \sin^2 x = 1$, so

$\cos^2 x = 1 - \sin^2 x$.

① $\Leftrightarrow 1 - \sin^2 x = 1 - 6\sin x + 9\sin^2 x$

$\Leftrightarrow 10\sin^2 x - 6\sin x = 0$

$\Leftrightarrow 5\sin^2 x - 3\sin x = 0$

$\Leftrightarrow \sin x\,[5\sin x - 3] = 0$

So either $\sin x = 0$

OR $5\sin x - 3 = 0$.

for $\sin x = 0$ x must be equal to 0 or π or 2π.

However, $x = \pi$ does not satisfy the original equation.

for $5\sin x - 3 = 0 \Leftrightarrow \sin x = \frac{3}{5}$

$x = (-1)^n \sin^{-1}\left(\frac{3}{5}\right) + n\pi$

$\sin^{-1}\left(\frac{3}{5}\right) \simeq 0.6435$ rad.

$n = 0$ gives $x \simeq 0.6435$ which does not satisfy the original equation

$n = 1$ gives $x \simeq 2.498$ which works.

$n = 2$ outside of the range $[0, 2\pi]$

So: $x = 0$, 2π or 2.498.

(b) $\sin 2x = 1 + \cos 2x$ *for* $0 \le x \le \pi$

we square both sides

$\sin^2 2x = (1 + \cos 2x)^2$

but $\sin^2 2x = 1 - \cos^2 2x$

so $1 - \cos^2 2x = 1 + 2\cos 2x + \cos^2 2x$

$\Leftrightarrow 2\cos^2 2x + 2\cos 2x = 0$

$\Leftrightarrow \cos 2x\,[1 + \cos 2x] = 0$

either $\cos 2x = 0$ OR $\cos 2x = -1$

Case 1 for $\cos 2x = 0 = \cos\frac{\pi}{2}$

$2x = \pm\frac{\pi}{2} + 2n\pi$ is the general solution

$\Leftrightarrow x = \pm\frac{\pi}{4} + n\pi$

$n = 0$ gives $x = \frac{\pi}{4}$ which satisfies the original equation

$n = 1$ gives $x = \frac{3\pi}{4}$ which does NOT

other values of n give values outside of $[0, \pi]$

Case 2 for $\cos 2x = -1$

$2x = \pm\pi + 2n\pi$

$\Leftrightarrow x = \pm\frac{\pi}{2} + n\pi$

$n = 0$ gives $x = \frac{\pi}{2}$ which satisfies the original equation

other values of n give values outside of $[0, \pi]$.

So two solutions

$x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$