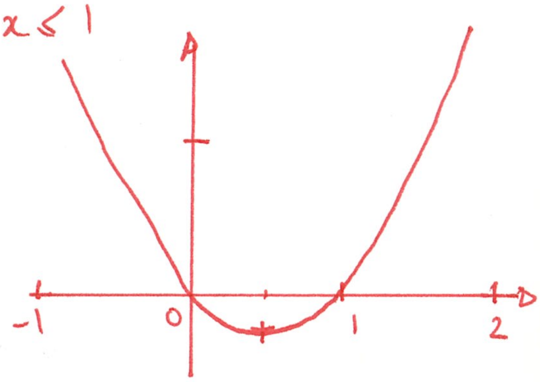
QUADRATIC INEQUALITIES

1. Solve the inequality $x(x-1) \leq 0$ The quadratic expression has 2 roots 0 and 1.

For $x \geq 0$, then $x-1 \leq 0$ for $x(x-1)$ to be negative.
i.e. $x \leq 1$ So $0 \leq x \leq 1$

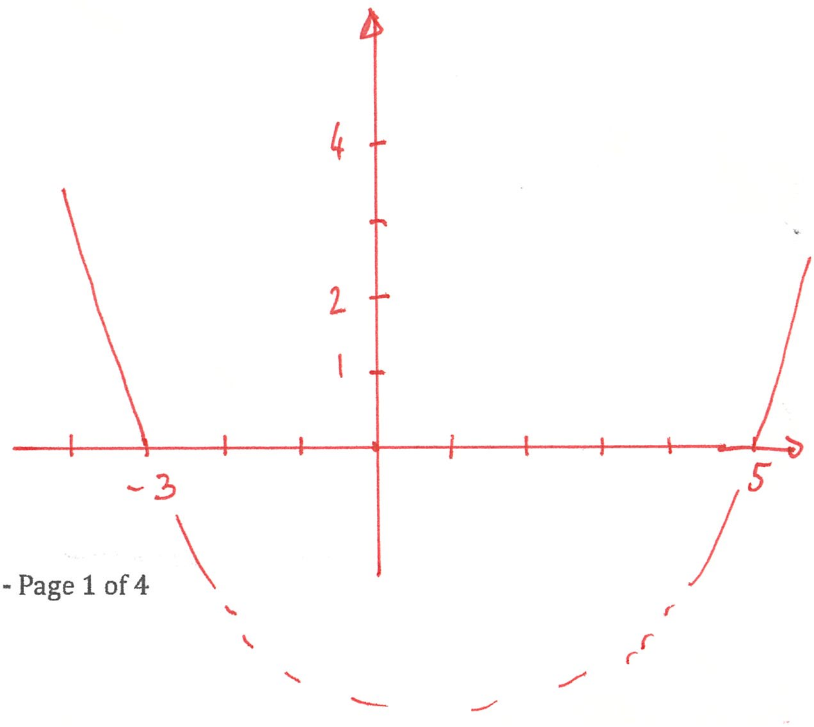
if $x \leq 0$ then $x-1$ must be positive, i.e $x \geq 1$
Both $x \leq 0$ and $x \geq 1$ exclude each other. So there is no possibility of $x \leq 0$.
$\therefore$ the only interval solution is $0 \leq x \leq 1$
which is visible on the graph of $f(x) = x(x-1)$



2. Solve the inequality $x^2 - 2x - 15 \leq 0$ $\Delta = b^2 - 4ac = 4 - 4 \times (-15) = 64 = 8^2$

So two roots $x_1 = \dfrac{2+8}{2} = 5$ and $x_2 = \dfrac{2-8}{2} = -3$
So $x^2 - 2x - 15 \leq 0$ is equivalent to $(x-5)(x+3) \leq 0$.
if $x \leq -3$ then we must have $x - 5 \geq 0$ i.e $x \geq 5$
Both are contradictory so this is not possible.
then $x + 3 \geq 0$, so $x - 5 \leq 0$ i.e. $x \geq -3$ and $x \leq 5$
So $-3 \leq x \leq 5$

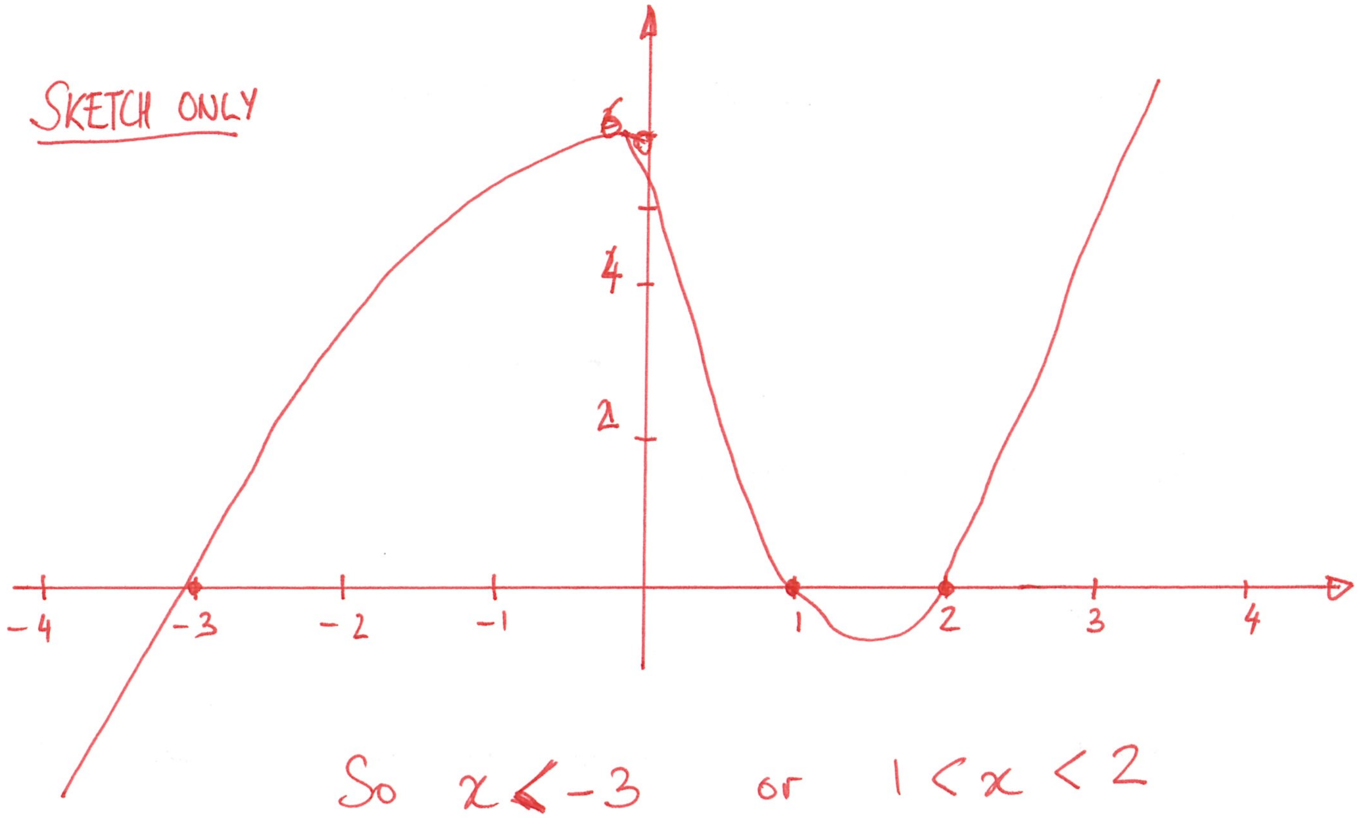
using a graphical method



QUADRATIC INEQUALITIES

3. Solve the inequality $(x-1)(x+3)(x-2) < 0$

SKETCH ONLY



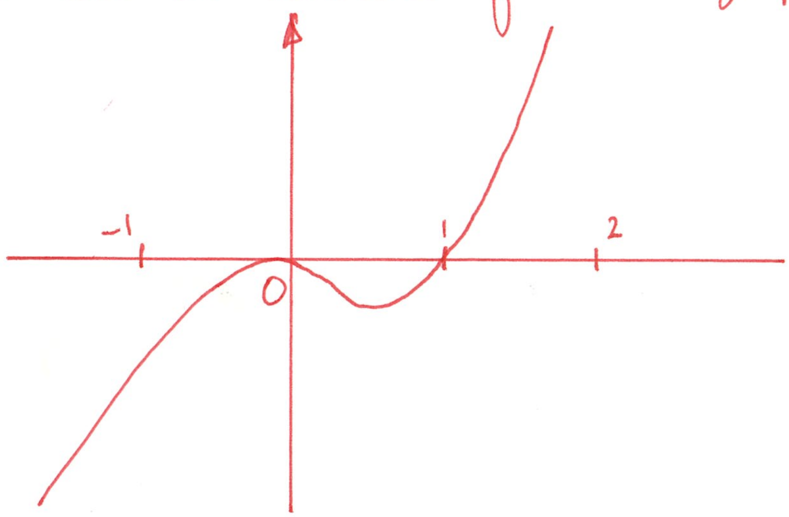
So $x < -3$ or $1 < x < 2$

4. Solve the inequality $x^2(x-1) \leq 0$

x^2 is always positive, so for $x^2(x-1)$ to be negative,

we must have $x-1 \leq 0$, i.e. $x \leq 1$

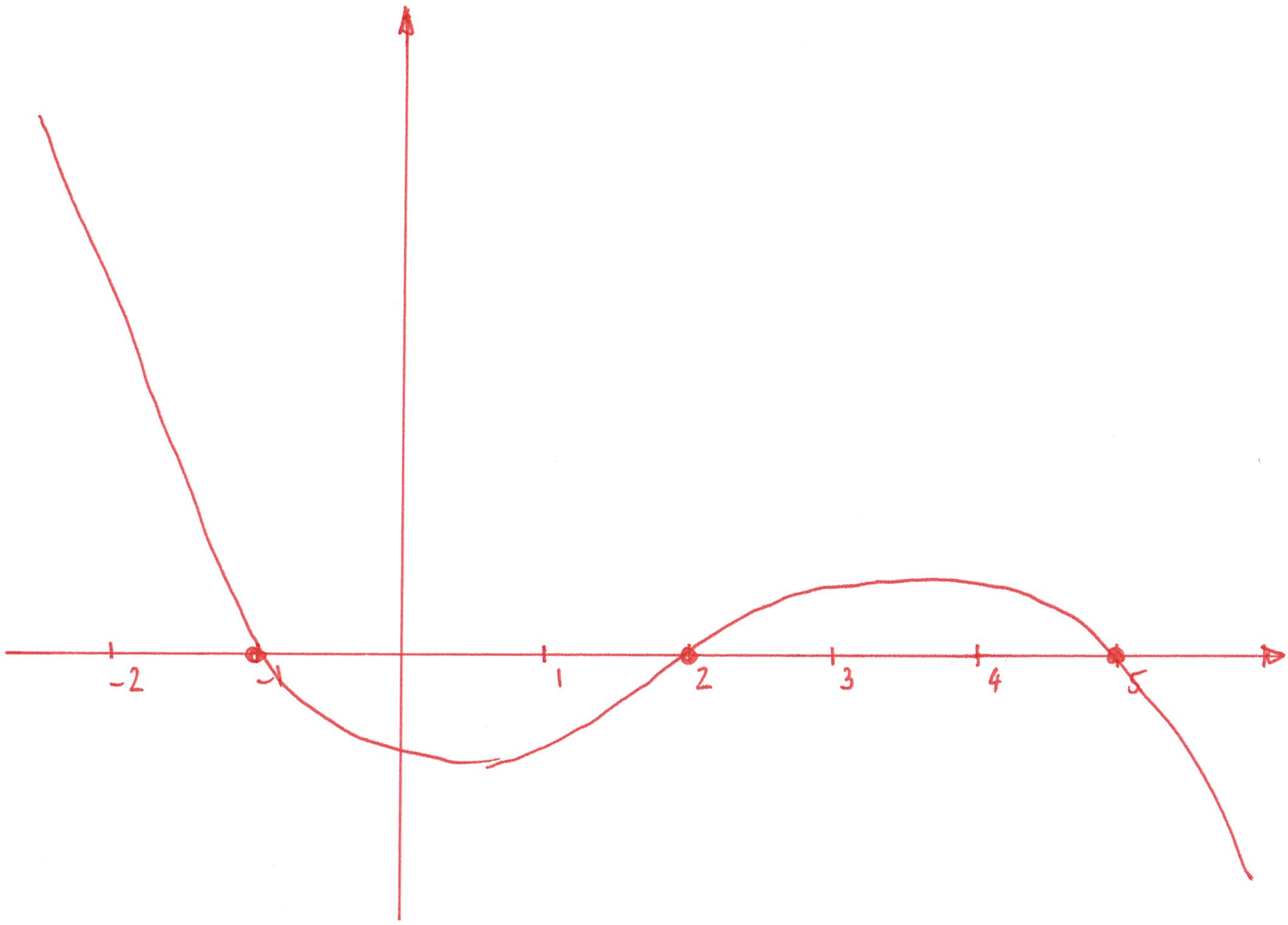
which can be deducted from the graph of $f(x) = x^2(x-1)$



QUADRATIC INEQUALITIES

5. Solve the inequality $(2-x)(x-5)(x+1) > 0$

The polynomial function $f(x) = (2-x)(x-5)(x+1)$ has 3 roots $x = 2$, $x = 5$ and $x = -1$



So for the expression to be positive, we must have either $x < -1$ or $2 < x < 5$

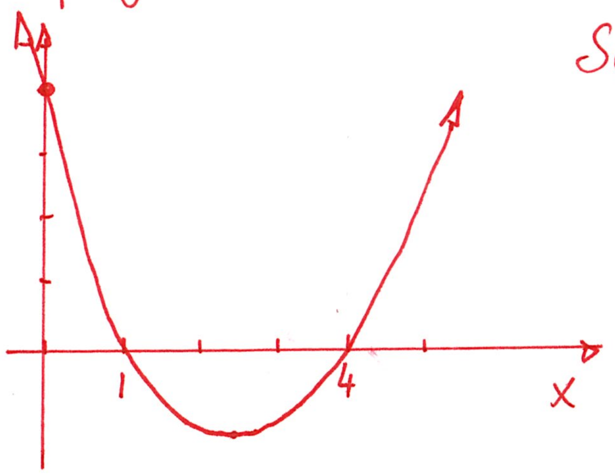
6. Solve the inequality $2^{2x} - 5(2^x) + 4 \leq 0$

We do a change of variable $X = 2^x$. So the inequality becomes

$X^2 - 5X + 4 \leqslant 0$.

This quadratic expression has 2 roots $X = 1$ and $X = 4$, so the inequality is equivalent to $(X-1)(X-4) \leqslant 0$

Graphing the parabola:



So we must have $1 \leqslant X \leqslant 4$

$\iff 1 \leqslant 2^x \leqslant 4$

But $1 = 2^0$ and $4 = 2^2$

So we must have $1 \leqslant x \leqslant 2$ which is the possible range of values

7. Solve the inequality $1 - x < 2x + 1 < x + 4$

First, we consider the inequality on the left-hand-side (LHS)

$1 - x < 2x + 1 \iff -x < 2x \iff 3x > 0 \iff x > 0$

Secondly, we consider the inequality on the right-hand-side (RHS)

$2x + 1 < x + 4 \iff x + 1 < 4 \iff x < 3$

For both to be valid at the same time, we must have: $0 < x < 3$