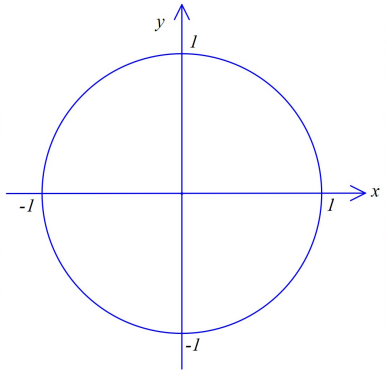
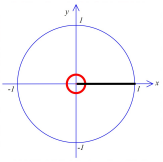
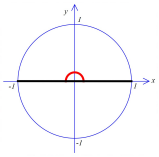
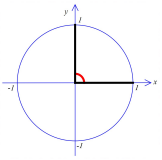
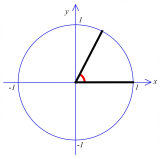
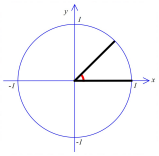
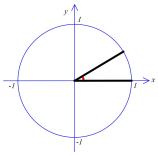
The radian as a unit to measure angles

The **radian** (abbreviation rad.) is the System International (SI) unit for measuring angles, and is the standard unit of angular measure used in advanced Mathematics, instead of degrees.

Using radians rather than degrees makes some calculations much easier.

The definition of a radian is based on the unit circle, which full circumference is 2π.



The length of a 360˚ arc is 2π. Therefore, an angle of 360˚ corresponds to 2π radians.	
The length of a 180˚ arc is π. Therefore, an angle of 180˚ corresponds to π radians.	
The length of a 90˚ arc is $\frac{\pi}{2}$ Therefore, an angle of 90˚ corresponds to $\frac{\pi}{2}$ radians.	
The length of a 60˚ arc is $\frac{\pi}{3}$ Therefore, an angle of 60˚ corresponds to $\frac{\pi}{3}$ radians.	
The length of a 45˚ arc is $\frac{\pi}{4}$ Therefore, an angle of 45˚ corresponds to $\frac{\pi}{4}$ radians.	
The length of a 30˚ arc is $\frac{\pi}{6}$ Therefore, an angle of 30˚ corresponds to $\frac{\pi}{6}$ radians.	

For the same angle, the formula to convert from degrees to radians is:

$$\frac{\boldsymbol{\theta_{in\ degrees}}}{\mathbf{360}} = \frac{\boldsymbol{\theta_{in\ radians}}}{\boldsymbol{2\pi}}$$

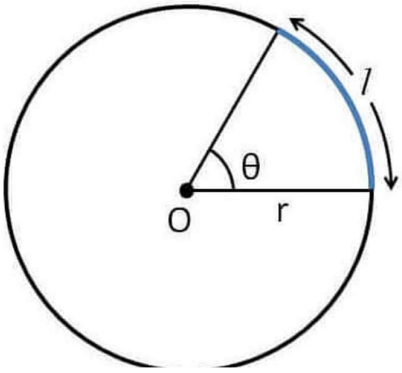
(the HSC formula sheet says: $\frac{\theta_{in\ degrees}}{180} = \frac{\theta_{in\ radians}}{\pi}$)

which can be rearranged as follows:

$$\theta_{in\ degrees} = \frac{360}{2\pi} \times \theta_{in\ radians} \qquad \theta_{in\ radians} = \frac{2\pi}{360} \times \theta_{in\ degrees}$$

The advantages of using radians instead of degrees are shown below:

Consider a circle of radius r



- **The length of an arc** is given by the formula: $l_{arc} = \frac{\theta_{in\ degrees}}{360} \times 2\pi r$

but as $\frac{\theta_{in\ degrees}}{360} = \frac{\theta_{in\ radians}}{2\pi}$, $l_{arc} = \frac{\theta_{in\ radians}}{2\pi} \times 2\pi r$

which simplifies as: $\boldsymbol{l_{arc} = \theta_{in\ radians} \times r}$ (a much simpler formula)

- **The area of a sector** is given by the formula: $A_{sector} = \frac{\theta_{in\ degrees}}{360} \times \pi r^2$

but as $\frac{\theta_{in\ degrees}}{360} = \frac{\theta_{in\ radians}}{2\pi}$, $A_{sector} = \frac{\theta_{in\ radians}}{2\pi} \times \pi r^2$

which simplifies as: $\boldsymbol{A_{sector} = \frac{\theta_{in\ radians}}{2} \times r^2}$