PROVING DIVISIBILITY BY INDUCTION

1 If k and M are integers, which of the following expressions does *not* always generate an integer?

A $9M + 4 \times 7k$ **B** $9M - 4 \times 7k$ **C** $9M \div 4 \times 7k$ **D** $9M \times 4 \times 7k$

Prove by induction

4 $3^n + 2^n$ is divisible by 5 for all odd integers $n \geq 1$.

5 $5^n + 2(11^n)$ is a multiple of 3 for all positive integers n.

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6 **(a)** Factorise $k(k+1)(k+2)+3(k+1)(k+2)$.

(b) Hence prove that $n(n+1)(n+2)$ is divisible by 3 for all positive integers n.

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7 $3^{3n} + 2^{n+2}$ is divisible by 5 for all positive integers n.

8 $7^n - 2^n$ is divisible by 9 for all even integers greater or equal to 2

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14 **(a)** Show that $(k+3)^3 = k^3 + 9k^2 + 27k + 27$.

(b) Hence prove that the sum of the cubes of three consecutive positive integers is divisible by 3.

15 Prove that the polynomial $(x-1)^{n+2}+x^{2n+1}$ is divisible by x^2-x+1 for all positive integers n.

(*Note:* In step 2, you can't say $(x-1)^{k+2}+x^{2k+1}=(x^2-x+1)M$ where M is an integer. You must say $(x-1)^{k+2}+x^{2k+1}=(x^2-x+1)M(x)$, where $M(x)$ is a polynomial, and continue this through the rest of the proof.)

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16 Prove that $x^n - 1$ is divisible by $x - 1$ for all positive integers n. (Use $x^{k+1} - 1 = x^{k+1} - x^k + x^k - 1$.)