PASCAL'S TRIANGLE - BINOMIAL THEOREM

1 Use Pascal's triangle to find the expansion of each of the following.

(a) $(1+x)^6$ (b) $(1+x)^8$ (c) $(1+b)^7$ (d) $(1+2x)^3$

a) $(1+x)^6 = \sum_{i=0}^{6} {}^6C_i\, x^i = 1 + {}^6C_1 x + {}^6C_2 x^2 + {}^6C_3 x^3 + {}^6C_4 x^4 + {}^6C_5 x^5 + x^6$

$= 1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$

b) $(1+x)^8 = 1 + {}^8C_1 x + {}^8C_2 x^2 + {}^8C_3 x^3 + {}^8C_4 x^4 + {}^8C_5 x^5 + {}^8C_6 x^6 + {}^8C_7 x^7 +$

$= 1 + 8x + 28x^2 + 56x^3 + 70x^4 + 56x^5 + 28x^6 + 8x^7 + x$

c) $(1+b)^7 = 1 + {}^7C_1 b + {}^7C_2 b^2 + {}^7C_3 b^3 + {}^7C_4 b^4 + {}^7C_5 b^5 + {}^7C_6 b^6 + b^7$

$= 1 + 7b + 21b^2 + 35b^3 + 35b^4 + 21b^5 + 7b^6 + b^7$

d) $(1+2x)^3 = 1 + {}^3C_1 (2x) + {}^3C_2 (2x)^2 + {}^3C_3 (2x)^3$

$= 1 + 3 \times 2x + 3 \times 4x^2 + 8x^3$

$= 1 + 6x + 12x^2 + 8x^3$

4 The coefficient of x^4 in the expansion of $\left(x-\sqrt{2}\right)^6$ is:

A −30 $\boxed{\text{B } 30}$ C −60 D 60

${}^6C_4\left(-\sqrt{2}\right)^2 = 30$

5 Find the fourth term in the expansion of: **(a)** $(1+x)^5$ **(b)** $(1-2a)^7$ **(c)** $\left(1+\frac{3x}{4}\right)^6$

degree 3

a) ${}^5C_3 = 10$ so $10x^3$

b) ${}^7C_3 \times (-2a)^3 = 35 \times (-8)a^3 = -280a^3$

c) ${}^6C_3 \times \left(\frac{3x}{4}\right)^3 = 20 \times \frac{27}{64} x^3 = \frac{135}{16} x^3$

7 Use the expansion of $\left(1-\frac{1}{x}\right)^n$ to find the value of each expression correct to four decimal places.

(a) $\left(\frac{9}{10}\right)^6$ **(b)** $\left(\frac{19}{20}\right)^5$ **(c)** $\left(\frac{99}{100}\right)^4$

a) $1-\frac{1}{x}=\frac{9}{10}$ so $\frac{1}{x}=1-\frac{9}{10}=\frac{1}{10}$ so $\boxed{x=10}$

$\left(\frac{9}{10}\right)^6=\left(1-\frac{1}{10}\right)^6 = 1+{}^6C_1\left(\frac{-1}{10}\right)^1+{}^6C_2\left(\frac{-1}{10}\right)^2+{}^6C_3\left(\frac{-1}{10}\right)^3+{}^6C_4\left(\frac{-1}{10}\right)^4+{}^6C_5\left(\frac{-1}{10}\right)^5+\left(\frac{-1}{10}\right)^6$

so $\left(\frac{9}{10}\right)^6 = 1-\frac{6}{10}+\frac{15}{10^2}-\frac{20}{10^3}+\frac{15}{10^4}-\frac{6}{10^5}+\frac{1}{10^6}$

$\left(\frac{9}{10}\right)^6 = 1-0.6+0.15-0.02+0.0015-0.00006+0.000001$

$\simeq 0.5314$

b) $\frac{19}{20}=1-\frac{1}{x}$ so $\frac{1}{x}=\frac{1}{20}$ $x=20$

$\left(\frac{19}{20}\right)^5=\left(1-\frac{1}{20}\right)^5 = 1+{}^5C_1\left(\frac{-1}{20}\right)+{}^5C_2\left(\frac{-1}{20}\right)^2+{}^5C_3\left(\frac{-1}{20}\right)^3+{}^5C_4\left(\frac{-1}{20}\right)^4+\left(\frac{-1}{20}\right)^5$

$\left(\frac{19}{20}\right)^5 = 1-5/20+10/400-10/8000+5/160{,}000-1/3{,}200{,}000 \simeq 0.7738$

c) $\left(\frac{99}{100}\right)^4=\left(1-\frac{1}{100}\right)^4 = 1+{}^4C_1\left(\frac{-1}{100}\right)+{}^4C_2\left(\frac{-1}{100}\right)^2+{}^4C_3\left(\frac{-1}{100}\right)^3+\left(\frac{-1}{100}\right)^4$

$\simeq 0.9606$

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2. Write the simplified expansion for each of the following:

$(2+\sqrt{2})^4 = {}^4C_0 2^4 (\sqrt{2})^0 + {}^4C_1 2^3 \sqrt{2} + {}^4C_2 2^2 \times (\sqrt{2})^2 + {}^4C_3 2 \times (\sqrt{2})^3 + {}^4C_4 2^0 (\sqrt{2})^4$

$= 2^4 + 4 \times 8 \times \sqrt{2} + 6 \times 4 \times 2 + 4 \times 2 \times 2\sqrt{2} + 4$

$= 68 + 48\sqrt{2}$

$(\sqrt{5}+\sqrt{3})^5 = {}^5C_0 (\sqrt{5})^5 + {}^5C_1 (\sqrt{5})^4 \sqrt{3} + {}^5C_2 (\sqrt{5})^3 (\sqrt{3})^2 + {}^5C_3 (\sqrt{5})^2 (\sqrt{3})^3 + {}^5C_4 (\sqrt{5})(\sqrt{3})^4 + (\sqrt{3})^5$

$= 25\sqrt{5} + 5 \times 25\sqrt{3} + 10 \times 5\sqrt{5} \times 3 + 10 \times 5 \times 3\sqrt{3} + 5\sqrt{5} \times 9 + 9\sqrt{3}$

$= [25 + 150 + 45]\sqrt{5} + [125 + 150 + 9]\sqrt{3}$

$= 220\sqrt{5} + 284\sqrt{3}$

4 The coefficient of x^5 in the expansion of $(2x+5)^8$ is:

A 32 000 **B** 4000 **C** 224 000 **D** 1792

${}^8C_5 (2x)^5 \times 5^3 = 56 \times 32 \times 125\ x^5 = 224{,}000$

5. Find the term independent of x in the expansions below:

$\left(x+\frac{1}{x}\right)^6$

$x^3 \times \frac{1}{x^3}$

So ${}^6C_3 (x)^3 \left(\frac{1}{x}\right)^3$

which is $\boxed{20}$

$\left(x+\frac{1}{x^2}\right)^6$

3rd term

$\downarrow$

x^6 $\quad$ $x^5 \times \frac{1}{x^2}$ $\quad$ $x^4 \left(\frac{1}{x^2}\right)$

So ${}^6C_2 = \boxed{15}$

5 The fourth term in the expansion of $\left(\frac{a}{b}-\frac{b}{a}\right)^6$ is:

A -20 B 20 C $\frac{15a^2}{b^2}$ D $-\frac{15b^2}{a^2}$

$${}^6C_3\left(\frac{a}{b}\right)^3\left(-\frac{b}{a}\right)^3 = 20 \times \left(\frac{a\times(-b)}{b\times a}\right)^3 = 20 \times (-1)^3 = -20$$

$\boxed{A}$

$\boxed{4}$ 6. Write and simplify the fourth term of $\left(\frac{m}{2}+3n\right)^8$

$${}^8C_3\left(\frac{m}{2}\right)^3(3n)^5 = 56\ \frac{m^3}{2^3} \times 243\, n^5$$

$$= 1701\, m^3 n^5$$

$\boxed{4}$ 8 Write and simplify the $(k+2)$-th term of $(a+b)^{2n}$.

$${}^{2n}C_{k+1}\ a^{2n-(k+1)}\ b^{k+1}$$

which is $\dfrac{(2n)!}{(2n-k-1)!\,(k+1)!} \times a^{2n-k-1} \times b^{k+1}$