FURTHER TRIGONOMETRY - REVISION (Cambridge)

1 Express the following angles in radians.

a $90°$ $\pi/2$ b $45°$ $\pi/4$ c $30°$ $\pi/6$ d $60°$ $\pi/3$

e $120°$ $2\pi/3$ f $150°$ $5\pi/6$ g $135°$ $3\pi/4$ h $225°$ $5\pi/4$

i $360°$ 2π j $300°$ $5\pi/3$ k $270°$ $3\pi/2$ l $210°$ $7\pi/6$

2 Express the following angles in degrees.

a π $180°$ b 2π $360°$ c 4π $720°$ d $\frac{\pi}{2}$ $90°$

e $\frac{\pi}{3}$ $60°$ f $\frac{\pi}{4}$ $45°$ g $\frac{2\pi}{3}$ $120°$ h $\frac{5\pi}{6}$ $150°$

i $\frac{3\pi}{4}$ $135°$ j $\frac{3\pi}{2}$ $270°$ k $\frac{4\pi}{3}$ $240°$ l $\frac{7\pi}{4}$ $315°$

m $\frac{11\pi}{6}$ $330°$

5 Find the exact value of:

a $\sin\frac{\pi}{6} = 1/2$ b $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ c $\cos\frac{\pi}{6} = \sqrt{3}/2$ d $\tan\frac{\pi}{3} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

e $\tan\frac{3\pi}{4} = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$ f $\cos\frac{5\pi}{3} = \frac{1}{2}$ g $\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$ h $\tan\frac{7\pi}{6} = \tan\frac{\pi}{6} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

7 Express these angles in radians in terms of π:

a $20°$ $\frac{\pi}{9}$ b $22.5°$ $\frac{\pi}{8}$ c $36°$ $\frac{\pi}{5}$ d $100°$ $\frac{5\pi}{9}$ e $112.5°$ $\frac{5\pi}{8}$ f $252°$ $\frac{7\pi}{5}$

8 Express these angles in degrees:

a $\frac{\pi}{12}$ $15°$ b $\frac{2\pi}{5}$ $72°$ c $\frac{20\pi}{9}$ $400°$ d $\frac{11\pi}{8}$ $247.5°$ e $\frac{17\pi}{10}$ $306°$ f $\frac{23\pi}{15}$ $276°$

9 a Find the complement of $\frac{\pi}{6}$. is $\frac{\pi}{2} - \frac{\pi}{6} = \pi/3$

b Find the supplement of $\frac{\pi}{6}$. is $\pi - \pi/6 = 5\pi/6$

10 Two angles of a triangle are $\frac{\pi}{3}$ and $\frac{2\pi}{9}$. Find, in radians, the third angle.

$\pi = x + \frac{\pi}{3} + \frac{2\pi}{9}$ so $x = \pi - \frac{\pi}{3} - \frac{2\pi}{9} = \frac{4\pi}{9}$

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1 Find, in radians, the acute angle θ that satisfies each equation.

a $\tan\theta = 1$

$\theta = \frac{\pi}{4}$

b $\sin\theta = \frac{1}{2}$

$\theta = \pi/6$

c $\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = \pi/4$

d $\tan\theta = \frac{1}{\sqrt{3}} = \frac{1/2}{\sqrt{3}/2}$

$\theta = \pi/6$

e $\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \pi/3$

f $\cos\theta = \frac{1}{2}$

$\theta = \pi/3$

3 Solve these equations for x over the domain $0 \le x \le 2\pi$:

a $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

b $\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

c $\tan x = -1$

$x = 3\pi/4$

or $x = 7\pi/4$

d $\sin x = 1$

$x = \pi/2$

e $2\cos x = \sqrt{3}$

$\cos x = \sqrt{3}/2$

$x = \pi/6, \frac{11\pi}{6}$

f $\sqrt{3}\tan x = 1$

$\tan x = \frac{1}{\sqrt{3}} = \frac{1/2}{\sqrt{3}/2}$

$x = \pi/6, \frac{7\pi}{6}$

g $\cos x + 1 = 0$

$\cos x = -1$

$x = \pi$

h $\sqrt{2}\sin x + 1 = 0$

$\sin x = -\frac{1}{\sqrt{2}}$

$x = \frac{5\pi}{4}, \frac{7\pi}{4}$

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4 Solve each equation for $0 \le \theta \le 2\pi$. Remember that a positive number has two square roots.

a $\sin^2\theta = 1$ **b** $\tan^2\theta = 1$ **c** $\cos^2\theta = \frac{1}{4}$ **d** $\cos^2\theta = \frac{3}{4}$

a) $\sin\theta = \pm 1$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

b) $\tan\theta = \pm 1$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

c) $\cos\theta = \pm \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

d) $\cos\theta = \pm \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

5 Consider the equation $\cos^2\theta - \cos\theta = 0$, for $0 \le \theta \le 2\pi$.

a Write the equation as a quadratic equation in u by letting $u = \cos\theta$.

b Solve the quadratic equation for u.

c Hence find the values of θ that satisfy the original equation.

a) $\Leftrightarrow \quad u^2 - u = 0 \quad \Leftrightarrow \quad u(u-1) = 0$ so either $u = 0$ or $u = 1$

$u = 0$ means $\cos\theta = 0$ so $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

$u = 1$ means $\cos\theta = 1$ so $\theta = 0$ or 2π

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6 Consider the equation $\tan^2\theta - \tan\theta - 2 = 0$, for $0 \le \theta \le 2\pi$.

a Write the equation as a quadratic equation in u by letting $u = \tan\theta$.

b Solve the quadratic equation for u.

c Hence find the values of θ that satisfy the original equation. Give the solutions correct to two decimal places where necessary.

a) $\Rightarrow u^2 - u - 2 = 0$ $\qquad \Delta = 1 - 4\times(-2) = 9 = 3^2$

$u = \frac{1 \pm 3}{2}$ so $u = 2$ or $u = -1$

if $u = 2$ then $\tan\theta = 2$ $\quad \theta = \arctan 2$

$\theta = 1.107$ rad or $\theta = \pi + 1.107 = 4.248$ rad

if $u = -1$ then $\tan\theta = -1$ $\quad \theta = \frac{3\pi}{4}$ or $\theta = \frac{7\pi}{4}$

7 Solve these equations for $0 \le \theta \le 2\pi$, by transforming each equation into a quadratic equation in u. Give your solutions correct to two decimal places where necessary.

a $\tan^2\theta + \tan\theta = 0$ (Let $u = \tan\theta$.)

$\tan\theta(\tan\theta + 1) = 0$

* for $\tan\theta = 0$ means

$\theta = 0, \pi, 2\pi$

* for $\tan\theta = -1$ means

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

b $2\sin^2\theta - \sin\theta = 0$ (Let $u = \sin\theta$.)

$\sin\theta(2\sin\theta - 1) = 0$

* for $\sin\theta = 0$ means

$\theta = 0$ or $\theta = \pi$ or $\theta = 2\pi$

* for $2\sin\theta - 1 = 0$ means

$\sin\theta = 1/2$

$\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

g $3\sin^2\theta + 8\sin\theta - 3 = 0$

$u = \sin\theta$

$3u^2 + 8u - 3 = 0$

$\Delta = 8^2 - 4\times(-3)\times 3 = 100$

$u = \dfrac{-8 \pm 10}{6}$

$u_1 = -3$ or $u_2 = \dfrac{1}{3}$

* $u_1 = -3$ means $\sin\theta = -3$ impossible

* $u_2 = 1/3$ means $\sin\theta = 1/3$

$\theta = \arcsin 1/3 \simeq 0.34$

or $\theta \simeq 2.80$

h $3\cos^2\theta - 8\cos\theta - 3 = 0$

$u = \cos\theta$

$3u^2 - 8u - 3 = 0$

$\Delta = (-8)^2 - 4\times(-3)\times 3 = 100$

$u = \dfrac{8 \pm 10}{6}$

$u_1 = 3$ or $u_2 = -\dfrac{1}{3}$

* $u_1 = 3$ means $\cos\theta = 3$ impossible

* $u_2 = -1/3$ means $\cos\theta = -1/3$

$\theta = \arccos(-1/3) \simeq 1.91$ rad

or $\theta = 4.37$

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9 Use the trigonometric identities from Chapter 5 to transform each equation so that it only involves one trigonometric function. Then solve it for $0 \le x \le 2\pi$. Give solutions correct to two decimal places where necessary.

a $2\sin^2 x + \cos x = 2$

b $\sec^2 x - 2\tan x - 4 = 0$

a) as $\sin^2 x + \cos^2 x = 1$, the equation becomes:

$2(1 - \cos^2 x) + \cos x = 2$

$\Leftrightarrow 2 - 2\cos^2 x + \cos x = 2$

$\Leftrightarrow -2\cos^2 x + \cos x = 0$

$\Leftrightarrow \cos x\,(1 - 2\cos x) = 0$

$\cos x = 0$ or $1 - 2\cos x = 0$

$x = \pi/2, 3\pi/2$ or $2\cos x = 1$

$x = \pi/3, 5\pi/3$

b) As $\sec^2 x = 1 + \tan^2 x$, the equation becomes:

$(1 + \tan^2 x) - 2\tan x - 4 = 0 \Leftrightarrow \tan^2 x - 2\tan x - 3 = 0$

$\Delta = (-2)^2 - 4 \times (-3) = 16 = 4^2$

So $\tan x = \dfrac{2 \pm 4}{2} = 3, -1$

if $\tan x = 3$ $\quad x = \arctan^{-1} 3 \simeq 1.25$ or $x \simeq 4.39$

if $\tan x = -1$ $\quad x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$